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Stencil mapping for lute bowl assembly

1. Introduction

The craft of the lute maker was neglected for about two centuries, from the end of the 18th century when the lute fell out of fashion until its revival in the 1970s. Hence today's lute makers are more dependent on historical references than are instrument makers working in an unbroken tradition such as that of the lute's close relative, the oud. They must rely on museums and private collections (Lundberg 1974; Cepelak 1999), and on the 'iconography', that is, on depictions of the lute in painting and sculpture.

As in other trades, the old masters kept their hard-won secrets to themselves, there are no detailed design/construction plans. However, there does exist an intriguing technical sketch by the 15th century polymath Arnaut de Zwolle, who was apparently interested in all manner of technical instruments including musical ones. His sketch (Fig.1) clearly illustrates a simple 'straight edge and compass' recipe for drawing the soundboard outline, arguably modified from a geometry used for medieval Arabic ouds (Rault 1999). In a recent article, Downing (2011) demonstrates that plausible geometric constructions can be similarly concocted for later lutes (and ouds), i.e., using only straight edge and compass.

In this note, the objective is to proceed from these geometrically constructed soundboard outlines to a mathematical description of the 2d shape of the ribs, that is, the strips of veneer from which the bowl of the lute is assembled. A set of shape templates for the ribs of the given bowl model is useful to the lute maker in the role of stencils from which to cut the ribs. However, it seems to be not widely appreciated that their shape can be plotted numerically. Instead, today's lute makers typically create their stencils in analog fashion by tacking tracing paper to the back of an instrument to be copied, or by filing down a blank *in situ* on a mould rack.

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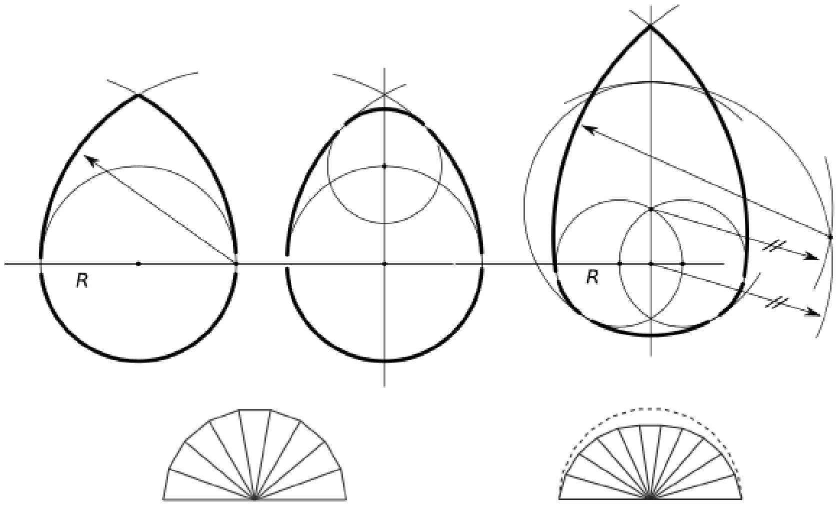

Figure 1. Belly elevations generated with compass and straight edge from a fiducial radius R, with associated seam structures in axial elevation. (L to r) 13th century Arabic; Arnaut de Zwolle (15th century); Magno Tieffenbrucker (1609)

There are two cases to consider. Zwolle specifies a *regular polygonal* cross-section, which implies that the ribs of his lute bowl were identical with one another. By the Baroque era, however, a shallower cross-section had become en vogue ('more flat in the back, they lie better on the stomach and do not endanger people to grow crooked' (Thurston 1958)). The shallow-bowl design the right of Fig.1 is from an extant lute dated 1607, by Magno Tieffenbrucker (Cepelak 1999). With this type the ribs are no longer identical, hence such bowls are known as 'multi-rib' bowls (Lundberg 1974).

In the following it is seen that the identical-rib case has a simple analytical form Eqn (2). The multi-rib case on the other hand requires a more involved computational treatment. The palmate frond of Fig.2 illustrates the generic look of a multi-rib stencil set computed via the method proposed.

2. Method

The general topic of paneling of surfaces into developable strips is very familiar in the computer-aided geometrical design (CAGD) literature (Weiss 1988; Hoschek 1998; Anastas et al. 2016), with many and diverse applications including e.g. architecture (Pottman et al. 2007; Postle 2012) and shoe design (Tang and Wang 2005). Another relevant strand in the CAGD literature looks at accommodating the shapes of classical instruments using various modeling curves, from cubic splines (Stroeker 2015) to more exotic cycloids or catenaries (Mann 2003). Here, because of the compass and straight edge mode of construction in Fig.1, we are concerned specifically with the older ancestral 'arc spline' type (Maier 2014).

2.1. *Sinusoidal approximation*

Let $\rho(z)$ denote the belly profile about its mid-axis z, composed of arc segments having the form $\rho(z) = (r^2 - (z - z_c)^2)^{1/2} + \Delta$, where r is a curvature radius and z_c, Δ locate the center of curvature.

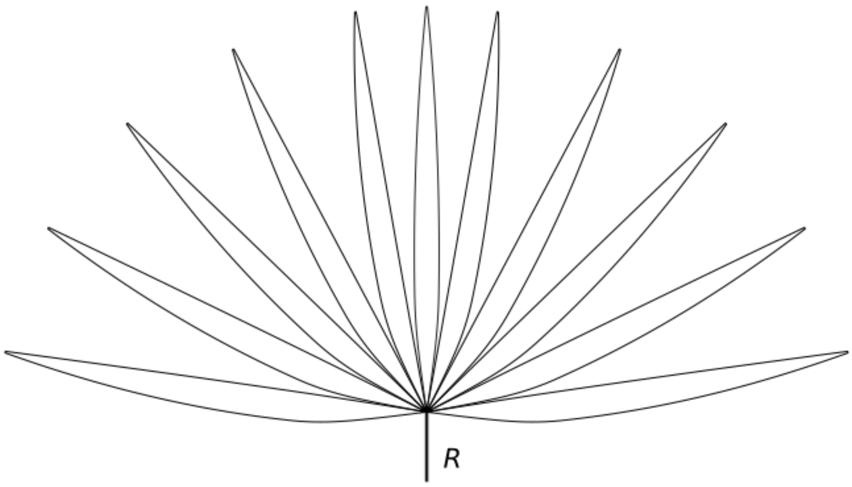

Figure 2. Algorithmically computed stencil map for the flattened-back 11-rib Tieffenbrucker bowl. Depth 143mm, breadth 353mm Cepelak (1999) ($\delta = 0.22$). The stem is the belly fiducial radius marked in Fig.1.

Table 1. Sinusoidal approximation parameters

segment	$u_0/\pi R$	r/R	ϕ/π	Δ/R
		Medieval Arabic		
A	0	1	0	0
B	1/2	2	1/4	-1
		Zwolle modification		
C	1	$2-2^{1/2}$	-0.957	0

Recasting $\rho(z)$ with respect to its path length u, one obtains the general form

$$\rho(u) = r\sin(u/r + \phi) + \Delta \tag{1}$$

Hence the stencil outline for identical-rib bowls is given by

$$\text{width}\,(u) \approx \pi\rho(u)/n, \tag{2}$$

where n is the rib count. The key assumption here is that the path length along a rib's mid-axis may be approximated by the path length along its edge. This approximation improves with increasing n.

Table 1 lists the coefficients of Eqn (1) for the two identical-rib designs of Fig.1. The tabulated u_0 values demarcate the crossover between segments of a profile occurs, i.e. the points of compound curvature.

2.2. *Multi-rib algorithm*

In so-called uv-mapping algorithms of the type available in standard CAGD packages, the 3d surface is discretized into triangular faces and these are then laid out in 2d according to some choice of edge-matching strategy. For non-developable surfaces (nonzero Gaussian curvature) edge matching can only ever be partially realized (Sheffer and de Sturler 2001). The mesh triangles of a developable surface on the other hand, by definition, may be arranged in a fully edge matched configuration (Postle 2012). Computation of lute rib stencils falls into this latter straightforward category.

The 3d bowl model to be mesh-discretized is fully specified by its seams, i.e., the loci along which adjacent ribs abut. Referring to the axial elevations of Fig.1 we assign each seam an index k, ranging from $k = 0$ at the mid-rib edges to $k = (n-1)/2$ at the bowl rim. It is assumed that n is odd, as is generally the case for lutes. With this indexing, the 'sweep phase' with respect to the belly normal is seen to be $(k+1/2)\pi/n$.

To accommodate flattening we introduce a parameter

$$\delta = \cos(\pi/2n)\,\frac{\text{breadth}/2}{\text{depth}} - 1 \tag{3}$$

and radially attenuate the seams by individual factors $f_k \leq 1$

$$f_k = \frac{1 + 2\delta k/(n-1)}{1+\delta}. \tag{4}$$

This completes specification of the 3d bowl model. Now let $\{L_i\}$, $\{M_i\}$ denote its discretization into 3d vertices along adjacent seams, i.e. the edges of a rib. Our objective is to map these seam pair vertices to the 2d pair $\{P_i\}$, $\{Q_i\}$ demarcating the corresponding stencil. This is achieved iteratively along the sequence in i, starting from $i = 0$, using the lengths shown in Fig.1 which are conserved under the mapping

$$\begin{aligned} a &= |L_i - M_i| \\ b &= |L_i - L_{i+1}| \\ c &= |L_{i+1} - M_i| \\ d &= |L_{i+1} - M_{i+1}| \\ e &= |M_i - M_{i+1}| \end{aligned} \tag{5}$$

The iterative step is

$$\begin{aligned} P_{i+1} &= P_i + (b/a)\mathbf{M}_\gamma(Q_i - P_i) \\ Q_{i+1} &= P_{i+1} + (d/c)\mathbf{M}_\epsilon(Q_i - P_{i+1}) \end{aligned} \tag{6}$$

where $\mathbf{M}$ is the anti-clockwise 2d rotation matrix for the angles γ, ϵ of Fig.3 determined via the cosine rule.

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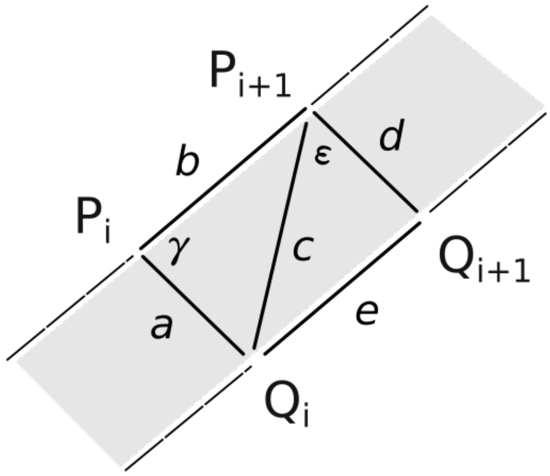

Figure 3. Definition of conserved lengths exploited by the multi-rib algorithm

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