The Ikhwan al-Safa Oud Revisited – Further to Comm. 1945 and some other thoughts.

The purpose of this Comm is to update and revise Comm 1945 with additional information concerning the design of an oud as recorded by the Ikhwan al-Safa (the Brethren of Purity) in their 5th Epistle on Music (for sources see Note 1)

The Ikhwan al-Safa were active around the 10th C and earlier in Basra, Iraq. The surviving manuscript copies of the Epistles date from between the late 12th C to the 15 C and so there is some variation in agreement between the manuscripts due to mistakes, adjustments, omissions and additions, the original texts having passed through the hands of several scribes over the course of the centuries.

The Brethren believed in one Divinity, a Universal harmony and the consequent significance of numbers and ratios.

Numbers and Ratios of Significance
Include for example:
Number 4 – the first perfect square, the four wind directions (North, South, East, West), the four seasons, the four elements (Fire, Air, Water and Earth), the four humours (Yellow Bile, Blood, Phlegm, Black Bile) and so on.
Number 6 - the first 'perfect' number, equal to the sum of its divisors (6 = 1 +2 +3). The next perfect number is 28 (28 = 1 +2 + 4 +7 +14).
Number 8 – the first cube number (2 x 2 x 2). After the sphere (and circle) the cubic form was considered to be the most eminent because of its property of equality (6 equal square faces, 8 equal plane angles, 12 parallel and equal intersections and 24 equal right angles. These four numbers 24:12:8:6 when applied to string lengths give the first four tones of the harmonic series.
Number 9 – the second perfect square.

Some significant ratios are 2:1; 3:2; 4:3; 5:4 and 9:8.

The Oud and Its Geometry
The proportions of a four course oud according to the Brethren are – "Its length should be half as much again as its width, its depth half its width and its neck a quarter of its (overall) length". So 2:1 is the ratio of body width to depth, 3:2 of body length to width and 4:3 overall length to body length.

That's it – no mention of soundhole diameter/location or the bridge position. Perhaps there was no soundhole (only small holes to vent the cavity of the body?). However, a 'conventional' soundhole is assumed here.

The revised proposed geometry (shown in Fig 1.) is based upon a circle of radius 8 arbitrary units. The overall length of the oud is then 32 units and the length of the neck is 8 units. The width of the neck joint is assumed to be 2 units. The upper sound board profile is defined by an arc of 22 units radius that intersects the vertical centre line at 7 units distance from the nut (this
point coincides with the fourth fret position given by the Brethren, ratio 4:3 string length (see below)
The bridge is located 4 units from the X axis i.e. midway between the X axis and the bottom of
the sound board or 1/6 the length of the sound board/body (Note 2)
The vibrating string length is then 28 units from nut to bridge.
A sound hole of 4 units diameter is placed with its centre 6 units from the X axis (Note 3).

**Physiology and Metrology**
The Brethren give some of the relative proportions of the perfect human body (i.e.of a newborn
child) as follows:

<table>
<thead>
<tr>
<th>Measurement</th>
<th>Proportion</th>
</tr>
</thead>
<tbody>
<tr>
<td>Height</td>
<td>8</td>
</tr>
<tr>
<td>Arms outstretched fingertip to fingertip</td>
<td>8</td>
</tr>
<tr>
<td>Soles of feet to knee</td>
<td>2</td>
</tr>
<tr>
<td>Knee to groin</td>
<td>2</td>
</tr>
<tr>
<td>Groin to top of heart</td>
<td>2</td>
</tr>
<tr>
<td>Top of heart to top of head</td>
<td>2</td>
</tr>
<tr>
<td>Elbow to fingertip</td>
<td>2</td>
</tr>
</tbody>
</table>

The unit here is the 'hand span' a distance measured from the tip of the thumb to tip of the little
finger with the fingers fully extended.

These are the same ideal proportions of the human body given by Leonardo da Vinci six
centuries later represented in his drawing of 'Vitruvian man'. The distance elbow to fingertip is a
cubit measure and a hand span is half a cubit.
The ancient cubit measure was further subdivided into 'finger' units or digits with 24 fingers to the 'cubit of man' (or the short common cubit) and 28 'fingers' to the 'Royal' or large cubit of the ancient Egyptians and Persians. The 'finger' unit standard measure is said to range from about 18.5 mm to 22.9 mm. based upon surviving measuring rods.

**Farmer's 'Finger'?**
The Ikhwan al-Safa did not assign an absolute physical measurement to an oud but it is of some interest to do so with reference to the ancient standards.

G.H. Farmer (Note 1) assumes a finger unit measuring 22.5 mm but without giving any indication of his source or the reason for his choice of unit. Further investigation suggests that Farmer may have been referring to the Arabic (black) cubit (said to date to the 9th C) having a value of just over 540 mm. It has been suggested that 'black' may also be translated as 'short' in Aramaic – so making it a common cubit of 24 fingers. Thus the Arabic (black) finger unit measures 22.5 mm.
Applying a finger unit measuring 22.5 mm to the proposed oud geometry gives a string length of 63 cm. (28 x 2.25) – conveniently comparable to the string length of modern Arabic ouds.

Alternatively could the actual physical width of the finger of a player have been used to determine the ideal dimensions of an oud to suit a player's physique? How is the width of a human finger measured in any case? Early Turkish accounts say it is the width of the index finger – placed upon a flat surface – and measured just below the finger nail.

My right hand index finger measures about 21 mm in width just below the fingernail. This translates into an ideal string length (for me) of 28 x 2.1 = 58.8 cm – so I should be playing a Turkish oud (man sized oud, string length 58.5 cm) rather than a longer string length Arabic oud. It so happens that the lute that I regularly play has a string length of 60 cm that I find a bit of a stretch at times. Just a coincidence perhaps?

The Fretting and Tuning of an Oud

The Brethren give the position of only four frets on the fingerboard – Pythagorean spacing with a full tone interval to the first fret – so ignoring the established practice of adding additional neutral or anterior frets (Al-Kindi, 9th C and Al-Farabi, 10th C – see Note 4).

The fourth fret (little finger fret) is located 1/4 total string length from the nut - the stopped string ratio 4:3 giving the sound of a ‘perfect fourth’. From this fret the adjacent open strings are tuned i.e. all of the four strings of an oud are tuned a fourth apart.

The position of the first fret (first finger fret) is found by dividing the string length into 9 equal parts – the first fret being on the ninth part nearest to the nut (9:8).

The distance from the first fret to the bridge is again divided into 9 equal parts and the third fret (ring finger fret) is located on the ninth part towards the nut.

The second fret (middle finger fret) is found by dividing the distance from fret 4 to the bridge into 8 equal parts. The second fret is then placed one division further towards the nut from fret 4.

Fig. 2, created to scale using dividers, shows the fretting layout (Note 5)

Note that the fourth fret position is 7 units (28/4) from the nut which is also the point where the arc defining the upper sound board profile intersects the Y axis of the proposed oud geometry.
The Strings of an Oud

The Brethren make the following general observation about vibrating strings for chordophones (with particular emphasis on the oud – “that most perfect of instruments”).

"When strings are identical in thickness, length and tension their sounds are identical. If identical in length (and tension?) but different in thickness the thinner strings will sound higher than the thicker ones. If they are identical in length and thickness but different in tension – the tenser strings will sound higher than the slacker ones."

(No mention is made about the influence of string linear density on string vibration suggesting perhaps that only one kind of string material and string density was used for instrument strings – at least for the oud?).

The oud has four strings of silk – no more and no less - each one being thicker than the next by the ratio 4:3 – the highest (pitched) string is called Zir, the second is Mathna, the third Mathlath and the thickest Bamm.

When tuning the oud the top string (Zir) is tensioned as high as it will stand without breaking (Note 6).

The second string (Mathna) is then stopped at the 4th fret (little finger fret) and tensioned to sound in unison with the open Zir string. Likewise the third string (Mathlath) is then stopped at the 4th fret and tuned in unison with the open Mathna string and also for the Bamm string – stopped again at the 4th fret and tuned in unison with the open Mathlath string. (So the strings are all tuned an interval of a fourth apart).

The Brethren then go on to state that the strings were made up from the following number of silk threads – Bamm 64, Mathlath 48, Mathna 36 and Zir 27. "When this is done, their lengths will be equal but their thicknesses (i.e. diameters) different according to the ratio 64:48:36:27" (i.e. 4:3 ratio increments)

Comment:

Silk strings are made from a bundle of individual filaments or threads twisted and glued together to form a uniform cylindrical string. Twisting a bundle of filaments results in a shortening of the original untwisted length and a consequent increase in the diameter (the volume remaining unchanged) – the greater the degree of twist, the greater the diameter increase of the completed string.

Twisting the filaments also reduces the tensile strength of a string. For maximum tensile strength the silk filament bundle making up the top string should have minimal twist. Information about silk instrument strings is limited but based upon a few preliminary trials, a simply twisted silk filament bundle will increase in diameter by about 7%, minimum twist and by about 18% to 22%, maximum twist (see Fig.3)

The ancient Chinese made their silk instrument strings as either three or four strand roped construction resulting in an increased diameter of about 29 % in the fully twisted state (calculated from a reduction in length of 40% of a fully twisted roped string - as recorded in the early Chinese texts on string making).

Assuming a string length of 63 cm and string material specific gravity of 1.3, the maximum pitch of the oud top string (tuned as high as it could go without frequent breaking) would be around f’
329 Hz (at an arbitrary A415 pitch standard). The remainder of the strings tuned a fourth apart would then be c' 247 Hz; g 185 Hz and d 139 Hz. For uniform plain cylindrical silk strings of specific gravity 1.3 - all at equal tension - and assuming a top string diameter of 0.45 mm, respective string diameters are then (4:3 ratio) f' - 0.45 mm, c' - 0.6 mm, g - 0.8 mm and d – 1.07 mm.

So far so good, however, it is not possible to achieve these diameters by simply twisting or even by roping the thread counts prescribed by the Brethren, the maximum diameter for a fourth plain twisted string being about 0.8 mm and a roped construction silk fourth string being about 0.84 mm. However, if string tensions are reduced then strings made according to the prescribed thread counts of the Brethren might still be (just about) feasible. Either all strings could be made simply twisted with tensions ranging from say 33N (Newtons) to 19N or with the first and second courses simply twisted and third and fourth strings of three strand and four strand construction (16 threads per strand) and tensions from 33N to 22 N.

So are the Brethren providing an important clue about the construction and tensioning of silk strings on a 10th C oud or are they (or the later copyists) just fixated on emphasising the significance of the 4:3 ratio?

Notes


2) As given by Arnault de Zwolle circa1450 ("stephanus stabit in sexta parte totius longitudinis …". 'A Fifteenth Century Lute Design', trans. Ian Harwood, L.S.J. #11 1960) and as found on some lutes of the 17th C (see 'On the Construction of the Lute Belly' Friedemann Hellwig, G.S.J. #21, March 1968)

3) This is the soundhole position found on the 19th C Al-Arja oud and 'Pythagoras of Ulm' lute – see Comms 1911 and 1945.


5) No doubt oud players of the 10th C used a similar process for laying out fret positions as described by Robert Dowland centuries later (‘Variety of Lute Lessons’, 1610) " take a thin flat ruler of whitish wood, and make it as long and straight as from the inward side of the nut to the inward side of the bridge ….. and take a payre of compasses ….."

6) Lute tuning practice for all gut stringing.

7) For a detailed discussion on the technical aspects of string design see Eph. Segerman's "A Primer on the History and Technology of Strings" at www.nrinstruments.demon.co.uk
PROPOSED IKHWAJAN AL-SATA
OUZ GEOMETRY - 10th C
4 COURSES - REV.1

SCALE: FINGER UNITS
0 - 4

TOTAL LENGTH = 32 UNITS
STRING LENGTH = 28 UNITS

R = 22 UNITS

DIAMETER = 4 UNITS
W = 16 UNITS

LITTLE FINGER
FRET POS.

Fig.1
THE IKhWAN AL-SAFA FRET POSITIONS

PATHAGOREAN SCALE

FRET #1 - FIRST FINGER (SABBABA)
FRET #2 - MIDDLE FINGER (WUSTA)
FRET #3 - RING FINGER (BINSIR)
FRET #4 - LITTLE FINGER (KHINSIR)

Fig. 2
\[ D = d \sqrt{\frac{l}{L}} \]

\[ d \text{ is calculated from total c/s area of filaments} \]

**Fig. 3**